

# ΘΕΜΑ Α

A1.  $\gamma$

A2.  $\delta$

A3.  $\delta$

A4.  $\beta$

A5. α λάθος

β. Σωστό

γ. λάθος

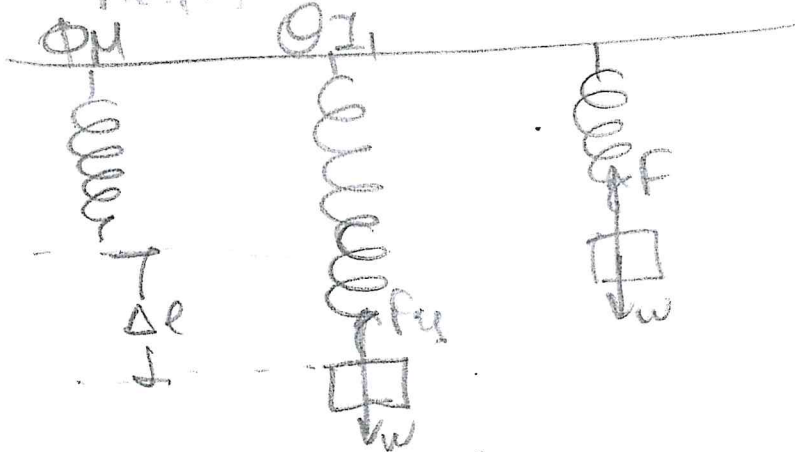
δ. Σωστό

ε. Σωστό

# ΘΕΜΑ Β

Πείρατα

Πείρατα 2

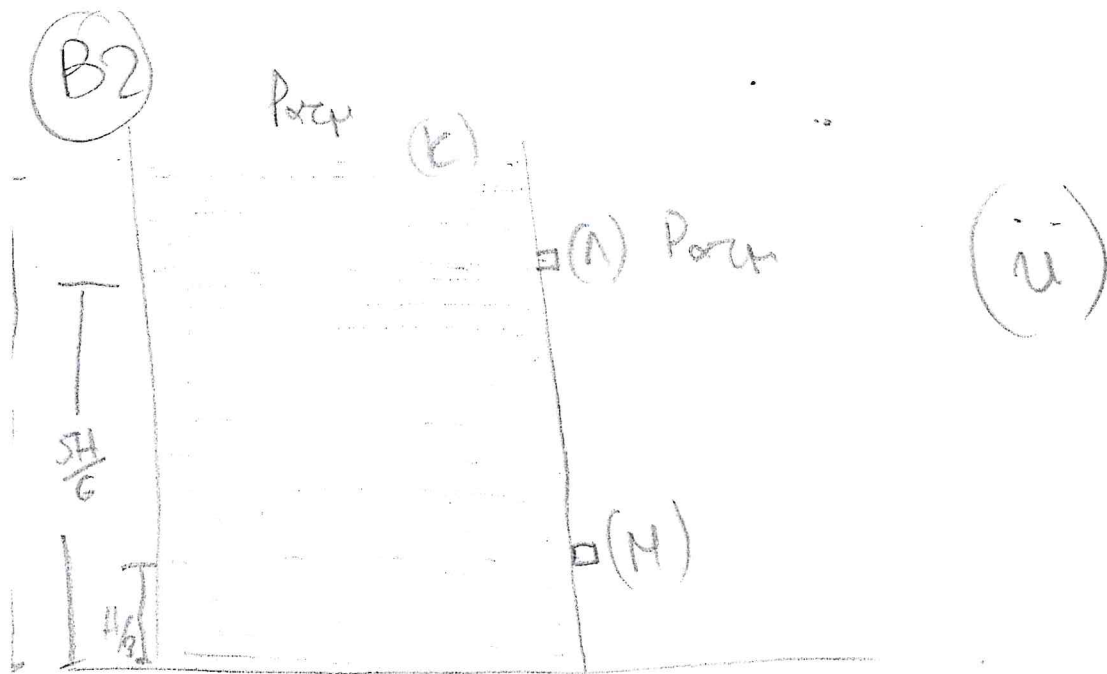


(i)

Σm ΘΙ :  $\Sigma F = 0 \Rightarrow W = F_{\text{ελ}} \Rightarrow mg = k\Delta\ell$

Πείρατα 1 :  $A_1 = \Delta\ell$

Πείρατα 2 : Σω ΦΜ :  $\Sigma F = 0$  γιατί  $F_{\text{ελ}} = 0$  κ  
 $\vec{F} = -\vec{W}$  άρα η ΝΘ.Ι είναι στο ΦΜ  
κ, το σύστημα ξεκινάει από τη ΘΙ άρα



$$\Pi_{\text{apx}} = \frac{V}{\Delta t_1} \quad \Pi_{\text{rey}} = \frac{V}{\Delta t_2}$$

$$\Pi_{\text{apx}} = A \cdot u_1$$

$$\Pi_{\text{rey}} = A u_1 + A u_2 = A(u_1 + u_2)$$

Bernoulli  $(k) \rightarrow (l)$ :

$$p_k + \rho p H + \frac{\rho u_k^2}{2} = p_l + \rho p \frac{5H}{6} + \frac{\rho u_1^2}{2} \Rightarrow$$

$$\rho p H - \frac{\rho p 5H}{6} = \frac{\rho u_1^2}{2} \Rightarrow \frac{\rho H}{6} = \frac{u_1^2}{2} \Rightarrow u_1 = \sqrt{\frac{gH}{3}}$$

Bernoulli  $(k) \rightarrow (m)$ :

$$p_k + \rho p H + \frac{\rho u_k^2}{2} = p_m + \frac{\rho p H}{3} + \frac{\rho u_2^2}{2} \Rightarrow$$

$$\rho p H - \frac{\rho p H}{3} = \frac{\rho u_2^2}{2} \Rightarrow \frac{2\rho H}{3} = \frac{u_2^2}{2} \Rightarrow u_2 = 2\sqrt{\frac{gH}{3}}$$

$$\frac{\Delta t_2}{\Delta t_1} = \frac{\frac{V}{\Pi_{\text{rey}}}}{\frac{V}{\Pi_{\text{apx}}}} = \frac{\Pi_{\text{apx}}}{\Pi_{\text{rey}}} = \frac{A u_1}{A(u_1 + u_2)} = \frac{\sqrt{\frac{gH}{3}}}{\sqrt{\frac{gH}{3}} + 2\sqrt{\frac{gH}{3}}} = \frac{1}{3}$$

B3|

$$P_1 = m_1 v_1$$

$$P_1' = m_1 v_1' = \frac{P_1}{5} \left. \vphantom{P_1'} \right\} \rightarrow v_1' = \frac{v_1}{5}$$

$$\frac{\Delta E}{E_1} \cdot 100\% = \frac{E_1' - E_1}{E_1} \cdot 100\% = \frac{\frac{1}{2} m_1 v_1'^2 - \frac{1}{2} m_1 v_1^2}{\frac{1}{2} m_1 v_1^2} \cdot 100\% =$$

$$= \frac{\left(\frac{v_1}{5}\right)^2 - v_1^2}{v_1^2} \cdot 100\% = \frac{\frac{v_1^2}{25} - v_1^2}{v_1^2} \cdot 100\% = -\frac{\frac{24}{25} v_1^2}{v_1^2} \cdot 100\%$$

$$= -\frac{24}{25} \cdot 100\% = -96\% \quad \text{aprox (ii)}$$

## ΘΕΜΑ Γ

$$\Gamma 1. \quad I = \frac{\mathcal{E}}{R_{\text{ολ}}}} = \frac{\mathcal{E}}{r + R_{\text{ext}}} = \frac{9}{1+2} = 3 \text{ A}$$

$$F_L = mg \Rightarrow BIL = 0,3 \cdot 10 \Rightarrow B \cdot 3 \cdot 1 = 3 \Rightarrow B = 1 \text{ T}$$

$$P_2 = V_2 \cdot I_2 \Rightarrow 6 = 6 \cdot I_2 \Rightarrow I_2 = 1 \text{ A}$$

$$V_2 = I_2 \cdot R_2 \Rightarrow 6 = 1 \cdot R_2 \Rightarrow R_2 = 6 \Omega$$

$$\frac{1}{R_{\text{ολ}}} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow R_{\text{ολ}} = 2 \Omega$$

$\Gamma 2.$  Επιταχύνεται λόγω βάρους. Στα άκρα του  $V_{\text{en}} = Bvl$  που αυξάνει, εκτοχευτεί  $I_{\text{en}} = \frac{\mathcal{E}_{\text{en}}}{R_{\text{ολ}}}$  που αυξάνει, δέχεται  $F_L = BI_{\text{en}}l$  που αυξάνει.

$$\text{Οα έχει } v_{\text{op}} \text{ ο } F_L = mg \Rightarrow BI_{\text{en}}l = mg \Rightarrow$$

$$B \cdot \frac{Bv_{\text{op}} \cdot l}{R_{\text{ολ}}} \cdot l = mg \Rightarrow v_{\text{op}} = \frac{mgR_{\text{ολ}}}{B^2 l^2} = \frac{3}{1} (R_{\text{ext}} + R_{\text{int}}) = 3(2+2)$$

$$v_{\text{op}} = 12 \text{ m/s}$$

$$\Gamma 3. \quad \frac{\Delta P}{\Delta t} = \Sigma F = mg - F_L = mg - \frac{B^2 l^2 v}{R_{\text{ext}} + R_{\text{int}}} = 1,5 \text{ N}$$

$$\Gamma 4. \quad I_{\text{en}} = \frac{Bvl}{R_{\text{ext}} + R_{\text{int}}} = \frac{1 \cdot 12 \cdot 1}{2+2} = 3 \text{ A} \quad V_2 = I_{\text{en}} \cdot R_{\text{ext}} = 6 \text{ V} \text{ ορα}$$

Αντικείμενο κανονισμένο

# ΘΕΜΑ Δ.

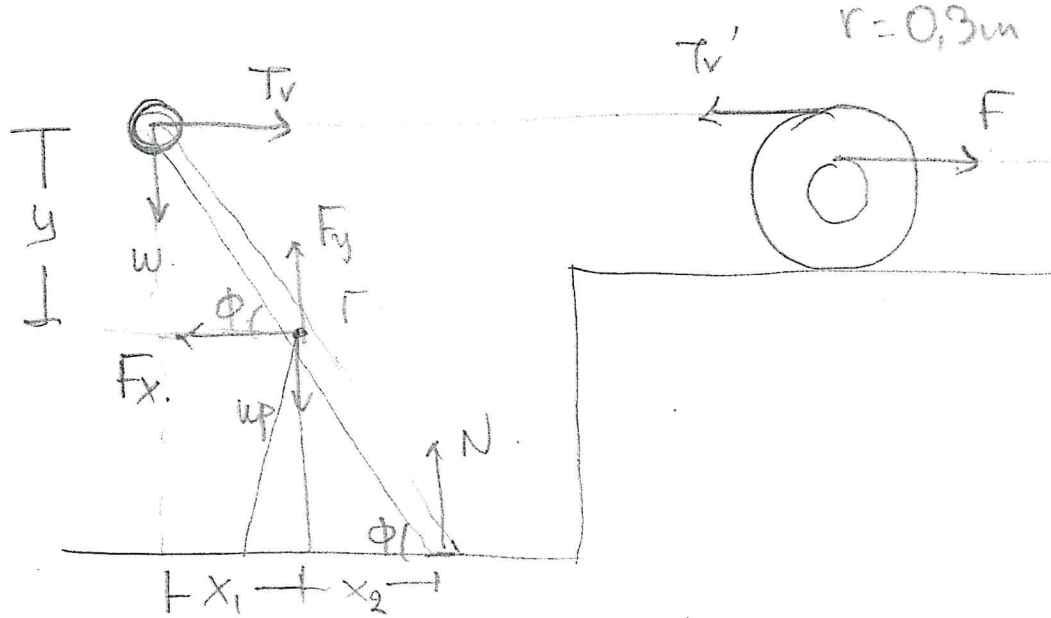
$$\eta + \phi = 0,8$$

Ράβδος:  $M_p = 3 \text{ kg}$   $l = 2 \text{ m}$   $I_{\text{cm}(p)} = \frac{M_p l^2}{12}$

$$\epsilon \omega \phi = 0,6$$

Σφαίριδιο:  $m = 1 \text{ kg}$

Τροχαλία:  $M_T = 7 \text{ kg}$   $R = 0,4 \text{ m}$   $I_{\text{cm}(T)} = \frac{M_T R^2}{2}$



1) Για το σύστημα ράβδος - σφαίριδιο έχουμε

$$\sum \tau_r = 0 \Rightarrow \vec{\tau}_w + \vec{\tau}_{T_v} + \vec{\tau}_N = 0 \Rightarrow$$

$$\tau_N + \tau_w = \tau_{T_v} \Rightarrow N \cdot x_2 + w \cdot x_1 = T_v \cdot y \quad (1)$$

$$\epsilon \omega \phi = \frac{x_2}{l/2} \Rightarrow x_2 = \frac{l}{2} \epsilon \omega \phi \quad (2)$$

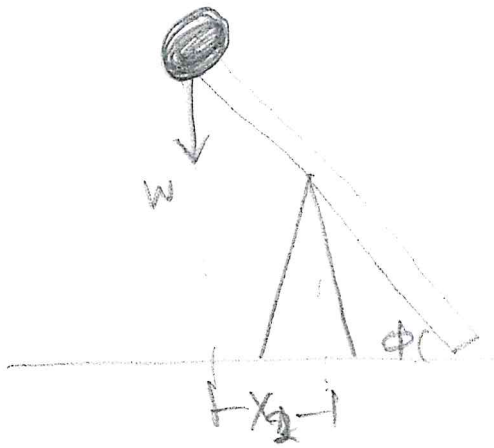
$$\epsilon \omega \phi = \frac{x_1}{l/2} \Rightarrow x_1 = \frac{l}{2} \epsilon \omega \phi \quad (3)$$

$$\eta + \phi = \frac{y}{l/2} \Rightarrow y = \frac{l}{2} \eta + \phi \quad (4)$$

$$(1), (2), (3), (4) \Rightarrow N \cdot \frac{l}{2} \epsilon \omega \phi + w \cdot \frac{l}{2} \epsilon \omega \phi = T_v \cdot \frac{l}{2} \eta + \phi \Rightarrow$$

$$N = \frac{T_v \cdot \eta + \phi - w \cdot \epsilon \omega \phi}{\epsilon \omega \phi} = \frac{10,5 \cdot \frac{8}{10} - 10 \cdot \frac{6}{10}}{\frac{8}{10} - \frac{6}{10}} = \frac{8,4 - 6}{0,2} = 4 \text{ N}$$

Δ2)



Αφού χάνεται η επαφή με το δάπεδο  $\perp: N=0$

Για το σύστημα ράβδος - σφαιρίδιο έχουμε:

$$\vec{\Sigma \tau_{\sigma\alpha\sigma\tau}} = I_{\sigma\alpha\sigma\tau} \vec{\alpha}_{\sigma\mu\nu}$$

$$I_{\sigma\alpha\sigma\tau} = I_{\text{cm}(r)} + I_{\text{εφα}(r)} = \frac{M r^2}{12} + m \frac{l^2}{4} =$$

$$= \frac{3 \cdot 2 \cdot 2}{12} + \frac{1 \cdot 2 \cdot 2}{4} = 1 + 1 = 2 \text{ kgm}^2$$

Τη στιγμή που κόβεται το νήμα και μόλις έχει χάσει η επαφή:

$$\vec{\Sigma \tau_{\sigma\alpha\sigma\tau}} = \vec{\tau}_w \Rightarrow \Sigma \tau_{\sigma\alpha\sigma\tau} = \tau_w = w \cdot x_1 = w \cdot \frac{l}{2} \text{ (αμφ =)}$$

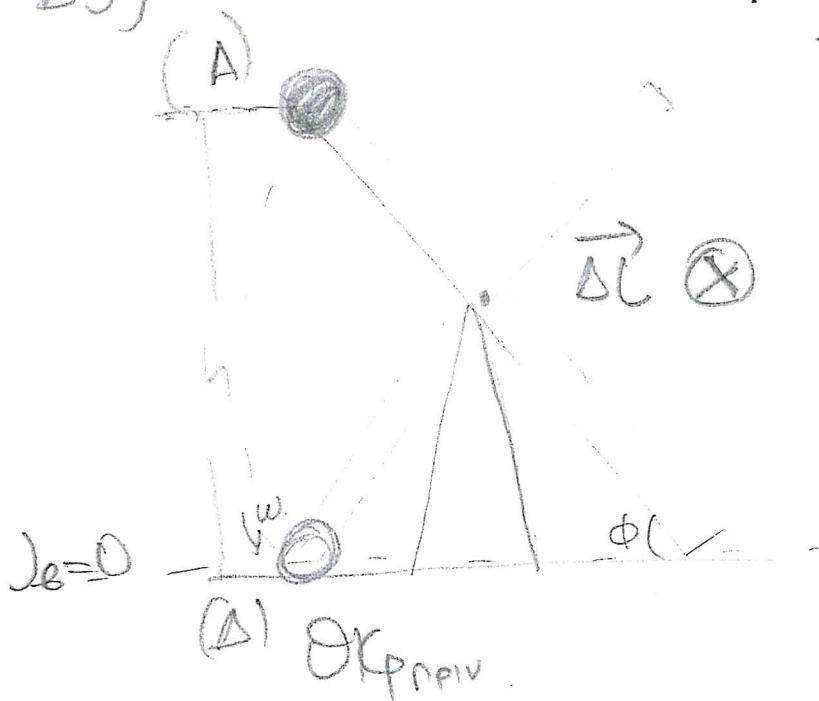
$$\Sigma \tau_{\sigma\alpha\sigma\tau} = 10 \cdot \frac{6}{10} = 6 \text{ Nm}$$

$$\text{Άρα } \alpha_{\sigma\mu\nu} = \frac{\Sigma \tau_{\sigma\alpha\sigma\tau}}{I_{\sigma\alpha\sigma\tau}} = \frac{6}{2} = 3 \text{ rad/s}^2$$

Για τη ράβδο τη στιγμή που χάνεται η επαφή:

$$\frac{\Delta L_F}{\Delta t} = \Sigma \tau_F = I_{\text{cm}(r)} \alpha_{\sigma\mu\nu} = \frac{M r^2}{12} \cdot \alpha_{\sigma\mu\nu} = \frac{3 \cdot 2 \cdot 2}{12} \cdot 3 = 3 \frac{\text{kgm}^2}{\text{s}^2}$$

Δ3)



$$m r \phi = \frac{h}{e} \Rightarrow h = l m r \phi = 2 \cdot \frac{8}{10} = 1,6 \text{ m}$$

Για την κίνηση του ελαττωτάτος πάβου-σφαιριδίου εφαρμόζατε ΑΔΜΕ από θέση (A) σε θέση (Δ).

$$E_{\text{μηχ}}(A) = E_{\text{μηχ}}(Δ) \Rightarrow K_A + U_A = K_Δ + U_Δ \quad (5)$$

$$K_A = 0 \quad (6) \text{ γιατί αρχικά ελαττωτά ακίνητο}$$

$$U_A = mgh + U_p \quad (7)$$

$$K_Δ = \frac{1}{2} I_{\text{ΟΚΡ}} \omega^2 \quad (8)$$

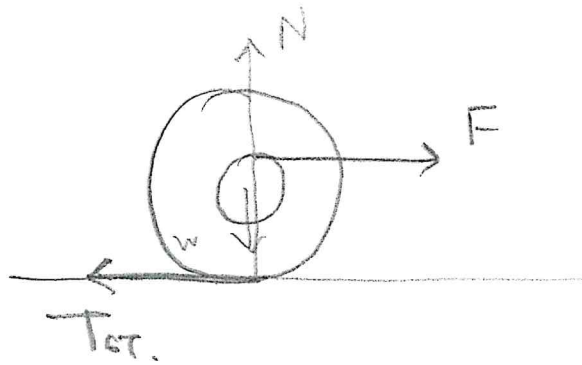
$$U_Δ = 0 + U_p \quad (9)$$

$$(5), (6), (7), (8), (9) \Rightarrow mgh + U_p = \frac{1}{2} I_{\text{ΟΚΡ}} \omega^2 + U_p \Rightarrow$$

$$mgh = \frac{1}{2} I_{\text{ΟΚΡ}} \omega^2 \Rightarrow \omega = \sqrt{\frac{2mgh}{I_{\text{ΟΚΡ}}}} = \sqrt{\frac{2 \cdot 1 \cdot 10 \cdot 1,6}{2}}$$

$$\omega = 4 \text{ rad/s}$$

Δ4.)



Για την τροχιά:

$$\text{Μεταφ: } \sum F_x = M_T \cdot a_{cm} \Rightarrow F - T_{στ} = M_T a_{cm} \quad (10)$$

$$\text{Στροφ: } \sum \tau_{cm} = I_{cm}(\sigma) \cdot \alpha_{\sigma} \Rightarrow \tau_F + \tau_{T_{στ}} = I_{cm}(\sigma) \cdot \alpha_{\sigma}$$

$$F \cdot r + T_{στ} \cdot R = \frac{M_T R^2}{2} \cdot \frac{a_{cm}}{R} \Rightarrow \frac{F \cdot r}{R} + T_{στ} = \frac{M_T a_{cm}}{2} \quad (11)$$

$$(10), (11) \Rightarrow F + \frac{F \cdot r}{R} = M_T \cdot a_{cm} + \frac{M_T \cdot a_{cm}}{2} \rightarrow$$

$$F + \frac{F \cdot r}{R} = \frac{3 M_T \cdot a_{cm}}{2} \Rightarrow \frac{7F}{4} = \frac{3 M_T \cdot a_{cm}}{2} \rightarrow$$

$$a_{cm} = \frac{7F}{6 M_T} = \frac{7 \cdot 12}{6 \cdot 7} = 2 \text{ m/s}^2$$

$$\Delta 5.) \quad X_{cm} = \frac{1}{2} a_{cm} \cdot \Delta t^2 = \frac{1}{2} \cdot 2 \cdot 2^2 = 4 \text{ m}$$

$$\theta = \frac{X_{cm}}{R} = \frac{40}{4} = 10 \text{ rad.}$$

$$W_{\text{τολ}} = F \cdot X_{cm} + \tau_F \cdot \theta = F \cdot X_{cm} + F \cdot r \theta = \\ = 19 (4 + \frac{3}{4} \cdot 10) = 84 \text{ Nm.}$$

$$\vec{\Delta L}_{\text{σφ}} = \vec{L}_{\text{σφ}}(\text{σφ}) - \vec{L}_{\text{ω}}(\text{σφ}) \Rightarrow$$

$$\Delta L_{\text{σφ}} = -L_{\text{σφ}}(\text{σφ}) - (+L_{\text{ω}}(\text{σφ})) =$$

$$= -I_{\text{σφ}} \omega - I_{\text{σφ}} \omega =$$

$$= -\frac{3}{2} I_{\text{σφ}} \omega = -\frac{3}{2} \cdot 2 \cdot 4 = -12 \frac{\text{kgm}^2}{\text{s}}$$

Άρα το μέτρο της μεταβολής  $\Delta L$  του  
συντηρητού ράβδου-σφαιριδίου είναι  $12 \text{kgm}^2/\text{s}$   
και φορά ομορροή της  $L_{\text{σφ}}(\text{σφ})$ .

